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FINAL REPORT FOR RESEARCH ON

INFINITE ELECTRICAL NETWORKS AND TRANSISTOR MODELING

Grant AFOSR-80-0205, 15 June 1979 to 14 September 1982

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### Research Objectives

The objectives of the research program were to develop still further the theory of infinite electrical networks and systems and to apply it to practical problems in engineering. Much of the research effort was directed toward the physical implications, computational methods, and practical applications of the theory. Of particular concern was the use of the theory in analyzing the behavior of a lateral bipolar transistor. Another goal was the development of an infinite-network analysis for the resistivity method of geophysical exploration.

#### Research Results

Both theoretical developments and practical applications were achieved during the three years that the grant was in force.

The first major theoretical result was the development of a method for computing the behavior of a grounded uniform semi-infinite grid, even when resistors, inductors, and capacitors appeared in the grid [1], [13]. No such general method existed previously. The basic idea was to represent the grid by a semi-infinite ladder network whose parameters are now operators on Hilbert's coordinate space. The uniformity of the grid is reflected in the uniformity of the operator ladder and allows thereby the use of the characteristic-impedance technique.

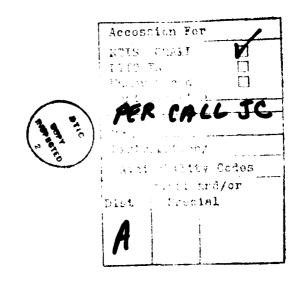
The next major result was the extension of our method to grounded grids that are not uniform. When that nonuniformity exists in only one direction, namely, perpendicular to the boundary of the semi-infinite grid, then the theory of continued fractions of operators can replace the characteristic-impedance technique to reestablish our computational method [4], [8], [11], [15]. Moreover, when the nonuniformity extends in all directions but is confined only to the grounding branches, another perturbational technique has been devised for the analysis of our grids [16].

Three other theoretical results were also achieved. A long-standing lacuna in the theory of infinite transmission lines was at last closed, and this finally established that theory on a completely rigorous basis [3]. A parametric representation for a very general kind of input-output linear time-varying system was devised [2], [9]. Finally, a new matroid arising from a certain class of infinite graphs was discovered [5].

We turn now to the practical applications of this theory. These applications arise from the fact that the partial-differential equation  $\nabla^2 \phi - c^2 \phi = g$  in the unkown  $\phi$  with  $c^2 > 0$  and g given, when approximated by finite differences, yields difference equations that can be modeled by a grounded electrical grid. In this case the sampled  $\phi$  values are the node voltages. So, when that equation is required to hold over a semi-infinite domain, we have at hand a semi-infinite grounded grid. Now, that equation describes a variety of physical phenomena. For example, it governs the minority-carrier density in the base region of a bipolar transistor. Moreover, when that transistor has a lateral configuration, its base region is effectively infinite in extent. Thus, our theory is immediately useful in analyzing the behavior of such transistors. This has been done in [6], [7], [14].

Another application arises from the partial-differential equation  $\nabla_{\bullet}(\sigma\nabla\varphi)=g$ , which governs, among other things, the electrical currents in a medium of variable conductivity  $\sigma$ ; here,  $\varphi$  is the electric potential. Now, the resistivity method of geophysical exploration makes use of such currents in the earth. Moreover, the earth appears as a semi-infinite medium to the geophysical explorer. Finally, the finite-difference approximation to this partial-differential equation can also be modeled by an electrical grid, but this time that grid is not grounded. Our theory can again be applied with however the complication that we now must work with unbounded operators, in contrast to the previous application where all operators are bounded. Nonetheless, a complete analysis was again achieved [12], [17], [18], and it led to some efficient computational methods of use to the geophysical explorer.

A survey of some of these results as well as the work of others is given in [8], which was an invited one-hour state-of-the-art lecture at the 1981 European Conference on Circuit Theory and Design.



#### **Publications**

The papers and publications that report the research supported by the grant are the following:

#### Papers in Refereed Journals

- 1. A.H. Zemanian, "The characteristic-resistance method for grounded semi-infinite grids," SIAM J. Math. Anal. vol. 12 (1981), pp. 115-138.
- 2. A. H. Zemanian, "Parametric representations of time-varying linear systems with Banach-space-valued signals," Systems and Control Letters, vol. 1 (1982), pp. 252-254.
- 3. A. H. Zemanian, "A justification of the characteristic-impedance method for lumped periodic cascades," <u>IEEE Trans. Circuits and Systems</u>, vol. CAS-29 (1982), pp. 323-327.
- 4. A. H. Zemanian, "Nonuniform semi-infinite grounded grids," SIAM J. Math. Anal., vol. 13 (Sept., 1982), in press.
- 5. A.C. Tucker and A.H. Zemanian, "A new matroid related to countably infinite, finitely chainlike graphs," Networks, in press. (The contribution of A.C. Tucker was supported by another grant).
- 6. Prasad Subramaniam and A.H. Zemanian, "Analysis of  $\nabla^2 \phi c^2 \phi = g$  in a semi-infinite medium with an irregular boundary by means of network manipulation," IEEE Trans. Circuits and Systems, submitted.

## Papers in Conference Proceedings

- 7. Prasad Subramaniam and A.H. Zemanian, "Simulation of a bipolar transistor by an infinite network," Proceedings of the 1980 IEEE International Conference on Circuits and Computers, Port Chester, N.Y., October 1-3, 1980, pp. 973-975.
- 8. A. H. Zemanian, "Nonuniform semi-infinite grounded grids," Proceedings of the 1981 International Symposium on Circuits and Systems, Chicago, Illinois, April 27-29, 1981, pp. 957-960.
- 9. A. H. Zemanian, "A parametric representation of time-varying linear systems with Banach-space-valued signals," 1981 International Symposium on the Mathematical Theoryof Networks and Systems, Santa Monica, California, August 5-7, 1981, pp. 275-277.
- 10. A. H. Zemanian, "The status of research on infinite electrical networks," Proceedings of the 1981 European Conference on Circuit Theory and Design, The Hague, The Netherlands, August 25-28, 1981.
- 11. A. H. Zemanian, "The finite power solution of a nonuniform semi-infinite grounded grid," Proceedings of the 1981 European Conference on Circuit Theory and Design, The Hague, The Netherlands, August 25-28, 1981.

12. A.H. Zemanian and Prasad Subramaniam, "The application of infinite electrical network theory to the geophysical exploration of layered strata," Proceedings of the Sixteenth Annual Conference on Information Sciences and Systems, Princeton University, Princeton, N.J., March 17-19, 1982.

### Technical Reports

(All published by the College of Engineering, State University of New York at Stony Brook).

- 13. A.H. Zemanian, The Characteristic-Resistance Method for Grounded Semi-infinite Grids, Tech. Rep. 330, August, 1979.
- 14. Prasad Subramaniam and A.H. Zemanian, Simulation of a Bipolar Transistor by an Infinite Network, Tech. Rep. 344, July, 1980.
- 15. A.H. Zemanian, Nonuniform Semi-infinite Grounded Grids, Tech. Rep. 345, August, 1980.
- 16. A.H. Zemanian and Prasad Subramaniam, A Numerical Solution of the Steady-State Heat-Radiation Equation in a Half-Plane or Half-Volume Using Network Manipulations, Tech. Rep. 369, August, 1981.
- 17. A.H. Zemanian and Prasad Subramaniam, A Theory for Ungrounded Electrical Grids and Its Applications to the Geophysical Exploration of Layered Strata: Part I, Tech. Rep. 373, September, 1981.
- 18. A.H. Zemanian and Prasad Subramaniam, A Theory for Ungrounded Electrical Grids and Its Applications to the Geophysical Exploration of Layered Strata: Part II, Tech. Rep. 380, October, 1981.

# Professional Personnel Associated with the Research Effort

A.H. Zemanian, Principal Investigator Prasad Subramaniam, Graduate Assistant Oscar Arenas, Graduate Assistant Tsu-Shuan Chang, Graduate Assistant Hsaio-Lin Chao, Graduate Assistant Nai Chi Lee, Graduate Assistant Kye Lim, Graduate Assistant Ram Subramaniam, Graduate Assistant Sen-Juan Tsai, Graduate Assistant

Prasad Subramaniam's doctoral thesis work was entirely supported by this grant. He received his Ph.D. in May, 1982; the title of his thesis was "Infinite Electrical Networks and Their Applications in Simulation and Modeling."

The other Graduate Assistants were used for short periods of time to help with the numerical computations related to the applications of our theoretical results.

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Infinite electrical networks; grounded grids; ungrounded grids; numerical simulation of transistors; numerical simulation for geophysical exploration; bipolar transistor behavior; the resistivity method; Laurent matrices; a new matroid on infinite graphs; operator networks; time-varying systems; applications of distribution theory.

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The theory and applications of infinite electrical networks was investigated. Methods were devised for computing the currents and voltages in uniform and non-uniform, grounded and ungrounded semi-infinite electrical grids. These results were applied to the numerical simulation of bipolar transistors and to the computational techniques arising in geophysical exploration. A complete and rigorous theory for infinite lumped transmission lines was at last achieved by closing a long-standing lacuna, and this was extended to ladder networks whose parameters are operators on a Hilbert space. A parametric representation for a general class of linear time-varying systems was devised. A new (CONTINUED)

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